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Flexible Boundary Method in Dynamic Substructure Techniques Including Different Component Damping

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There are various condensation methods for substructure techniques in structural dynamics. A generalized condensation method that comprises most of the classical condensation techniques and allows for arbitrary mode shapes within a standardized approach is described here. Within the framework of this method, a flexible boundary method is introduced that allows for elastic and mass-loaded boundaries for eigenmode determination, as well as for mode shapes that reflect the influence of damping. The relationship to other approaches taken from existing literature is examined. For damped structures, the flexible boundary method provides a condensation process that takes into account the influence of the complex eigenmodes of structures with nonproportional and high damping. To couple substructures with different component damping, the equivalent structural damping approach is provided. The problems associated with diagonal system damping of substructures and the full triple matrix product are overcome.

Nomenclature

$[A]$	=	attachment modes
$[B]$	=	flexibility matrix
$[C]$	=	structural damping matrix
$[D]$	=	viscous damping matrix
$\{F\}$	=	external force vector
$[G]$	=	condensation matrix
$[G^*]$	=	matrix of Ritz vectors
$[K]$	=	stiffness matrix
$[M]$	=	mass matrix
$[P]$	=	projection matrix
$\{q\}$	=	vector of generalized degrees of freedom
$[R]$	=	residual flexibility
$[T]$	=	transformation matrix
$\{x\}$	=	vector of physical displacements
$[\theta]$	=	complex eigenmodes
$[\Phi]$	=	static modes
$[\varphi]$	=	real eigenmodes
Ω	=	frequency domain variable
ω	=	eigenfrequency

Introduction

FOR structural dynamic investigations, large structures are usually divided into several substructures. For undamped structures, condensation and coupling techniques are already well known and established. Usually, the condensation of mathematical models is performed by using structural mode shapes under certain boundary conditions of the interfaces (I/Fs). Clamped boundaries are used for the Craig–Bampton [1] approach that is in wide use. Several other approaches extend the base of the mode shapes and will be discussed within this paper. A discussion of general topics concerning computational methods may be found in [2,3]. The first description of a generalized method that allows the investigation of several approaches within a unified framework is presented in [4].

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For structures with different component damping, severe problems occur, as described by Blelloch and Tengler [5]. Using the results of space shuttle analyses, they demonstrated that the combination of 1% damped modes with 2% damped modes on the substructure level yields several modes of the coupled structure with less than 1% and more than 2% damping. Moreover, the commonly used damping approach on the substructure level results in severe errors in the response analysis.

Several approaches were implemented in order to overcome these problems. The full triple matrix product (FTMP) is state of the art; several improvements (two-step method, Hruda–Benfield [5]) were investigated, but none of them is satisfying. In [6], the reasons for the problems are investigated. An approach that is based on the formulation of equivalent structural damping of substructures (ESD approach) is described.

Within this paper, a consistent formulation of the condensation, coupling, and analysis of structures with different component damping will be presented.

Ritz Approximation by Generalized Coordinates

In the frequency domain, the equation of motion of a substructure is noted as

$$-\Omega^2[M_{ff}]\{\tilde{x}_f\} + i\Omega[D_{ff}]\{\tilde{x}_f\} + ([K_{ff}] + i[C_{ff}])\{\tilde{x}_f\} = \{F_f(\Omega)\} \tag{1}$$

where $[M]$ denotes the mass matrix, $[K]$ denotes the stiffness matrix, $[D]$ denotes the viscous damping matrix, and $[C]$ denotes the damping matrix associated with structural damping. The displacement vector is $\{\tilde{x}\}$, and $\{F\}$ is the corresponding force vector. The f-set degrees of freedom (DOFs) include the I/F DOFs (i.e., at least the DOFs that will be coupled with other structures) and all the other independent DOFs of the substructure.

A condensation of the f-set DOFs to the reduced a set of the generalized coordinates $\{q_a^*\}$ is defined by the introduction of the approximation vector $\{x_f\}$:

$$\{x_f\} = [G_{fa}^*]\{q_a^*\} \tag{2}$$

where the number of the DOFs of the a set is smaller than the number of the f-set DOFs. The columns of the matrix $[G_{fa}^*]$ must be linear independent in order to avoid a rank deficiency problem. Apart from that, the matrix $[G^*_{fa}]$ is an arbitrary matrix. Any solution for the displacement vector $\{x_f\}$ will be a linear combination of the columns of the matrix $[G^*_{fa}]$.

Introducing the approximation $\{x_f\}$ from Eq. (2) into Eq. (1) and left multiplying with $[G^*_{fa}]^T$ yields

$$-\Omega^2[G^*_{fa}]^T[M_{ff}][G^*_{fa}]\{q^*_a\} + i\Omega[G^*_{fa}]^T[D_{ff}][G^*_{fa}]\{q^*_a\} + [G^*_{fa}]^T([K_{ff}] + i[C_{ff}])[G^*_{fa}]\{q^*_a\} = [G^*_{fa}]^T\{F_f(\Omega)\} \tag{3}$$

This equation results in a minimization of the estimation error in terms of work. Therefore, the vectors of the matrix $[G^*_{fa}]$ are referred to as Ritz vectors.

The physical coupling DOFs of the substructures that represent the displacements at the I/Fs are preserved in Eq. (3) only by certain condensation techniques with a proper choice of the Ritz vectors. In general, a consequence of an unrestrained choice of the Ritz vectors is the replacement of all the physical DOFs by generalized DOFs. To avoid the generalization of certain DOFs, a transformation of the condensation matrix is necessary. This transformation will be performed by a nonsingular quadratic matrix in order to leave the mathematical subspace of the initial chosen Ritz vectors unchanged. First, the f set will be partitioned into the t set that contains all the DOFs to be preserved (at least all the coupling DOFs) and the o set with all the other DOFs:

$$\{x_f\} = \begin{Bmatrix} x_t \\ x_o \end{Bmatrix} = \begin{bmatrix} G^*_{tt} & G^*_{tq} \\ G^*_{ot} & G^*_{oq} \end{bmatrix} \begin{Bmatrix} q^*_t \\ q^*_q \end{Bmatrix} \tag{4}$$

The partitioning of the a set of the generalized DOFs $\{q^*_a\}$ into the t set and the q set is arbitrary. It should be performed in such a way that the inverse of the matrix $[G^*_{tt}]$ exists. This requires that the t-set parts of the first t Ritz vectors are linear independent and are capable of representing arbitrary t-set displacements. Now, a transformation matrix $[T_{aa}]$ is defined:

$$[T_{aa}] = \begin{bmatrix} G^{*-1}_{tt} & -G^{*-1}_{tt}G^*_{tq} \\ 0_{qt} & I_{qq} \end{bmatrix} \tag{5}$$

The generalized DOFs $\{q^*_a\}$ will be replaced by the new DOFs $\{q_a\}$ with

$$\{q^*_a\} = [T_{aa}]\{q_a\} \tag{6}$$

Replacing $\{q^*_a\}$ in Eq. (2) yields

$$\{x_f\} = [G_{fa}]\{q_a\} \tag{7}$$

with

$$[G_{fa}] = [G^*_{fa}][T_{aa}] = \begin{bmatrix} I_{tt} & 0_{tq} \\ G^*_{ot}G^{*-1}_{tt} & G^*_{oq} - G^*_{ot}G^{*-1}_{tt}G^*_{tq} \end{bmatrix} \tag{8}$$

Within the new generalized vector $\{q_a\}$, the coupling DOFs of the t set are the preserved physical (not generalized) DOFs: $\{q_t\} = \{x_t\}$. This becomes obvious by the first row of the partitioned formulation.

Now, the condensed equation of motion with the preserved physical DOFs of the t set becomes

$$-\Omega^2[G_{fa}]^T[M_{ff}][G_{fa}]\{q_a\} + i\Omega[G_{fa}]^T[D_{ff}][G_{fa}]\{q_a\} + [G_{fa}]^T([K_{ff}] + i[C_{ff}])[G_{fa}]\{q_a\} = [G_{fa}]^T\{F_f(\Omega)\} \tag{9}$$

This very simple generalized condensation approach may be summarized by three steps. The first step is the choice of an arbitrary set of Ritz vectors and the establishment of $[G^*_{fa}]$. The second step is the transformation of $[G^*_{fa}]$ with $[T_{aa}]$. The third step is the condensation with $[G_{fa}]$.

To make any known condensation method available within the generalized approach, it is sufficient to assemble the Ritz vectors associated with that method within the matrix $[G^*_{fa}]$. A special aspect linked to the generalized condensation method is the avoidance of the rank deficiency problem.

Rank Deficiency Problem

Usually, the phenomenon of rank deficiency is associated with the increase of the number of zero eigenvalues of the unconstrained system matrices during the condensation process.

The choice of the columns of the matrix $[G^*_{fa}]$, the Ritz vectors, is arbitrary. Rank deficiency of the condensed mathematical model occurs, if the Ritz vectors are linear dependent. For practical applications, it is sufficient to ensure by a Cholesky factorization of the matrix $[G^*_{fa}]^T\ [G^*_{fa}]$ that all the columns of the matrix $[G^*_{fa}]$ are linear independent. If the Cholesky factorization fails, the transformation based on $[G^*_{fa}]$ will yield a rank-deficient problem. If the Ritz vectors are inherently linear independent (e.g., for the Craig–Bampton approach), this test is not necessary.

If linear dependent vectors exist, the linear dependency must be removed. Matrix $[\bar{G}^*_{fa}]$ shall be the original matrix of Ritz vectors with m columns that will yield a rank-deficient problem. Of these columns, n are linear independent with $m \geq n$. A linear independent matrix $[G^*_{fa}]$ with n columns is given by

$$[G^*_{fa}] = [\bar{G}^*_{fa}][X_n] \tag{10}$$

where $[X_n]$ is the matrix of the eigenvectors $\{x_i\}$ of the matrix $[\bar{G}^*_{fa}]^T[\bar{G}^*_{fa}]$ with eigenvalues $\lambda_i \neq 0$. The rank deficiency $m - n$ equals the number of eigenvalues $\lambda_i = 0$.

Flexible Boundary Method

A flexible boundary method will be introduced that allows for elastic and mass-loaded boundaries for the determination of the Ritz vectors. The flexible boundary method (F/B) demands for the constraint modes $[\Phi_{ot}]$ associated with the t set and a matrix $[\varphi_{fq}]$ that may be assembled by modes of the undamped system with arbitrary I/F conditions or any other suitable set of modes:

$$[G^*_{fa}]_{\mathrm{F/B}} = \begin{bmatrix} G^*_{tt} & G^*_{tq} \\ G^*_{ot} & G^*_{oq} \end{bmatrix} = \begin{bmatrix} I_{tt} & \varphi_{tq} \\ \Phi_{ot} & \varphi_{oq} \end{bmatrix} \tag{11}$$

The constraint modes are defined for unit displacements at the t-set DOFs:

$$\begin{bmatrix} K_{tt} & K_{to} \\ K_{ot} & K_{oo} \end{bmatrix} \begin{bmatrix} I_{tt} \\ \Phi_{ot} \end{bmatrix} = \begin{bmatrix} F_{tt} \\ 0_{ot} \end{bmatrix} \tag{12}$$

Therefore,

$$[\Phi_{ot}] = -[K_{oo}]^{-1}[K_{ot}] \tag{13}$$

Because $[G^*_{tt}]$ is the identity matrix, Eq. (8) simplifies to

$$[G_{fa}]_{\mathrm{F/B}} = \begin{bmatrix} I_{tt} & 0_{tq} \\ G^*_{ot} & G^*_{oq} - G^*_{ot}G^*_{tq} \end{bmatrix} = \begin{bmatrix} I_{tt} & 0_{tq} \\ \Phi_{ot} & \varphi_{oq} - \Phi_{ot}\varphi_{tq} \end{bmatrix} \tag{14}$$

Modes of the structure with elastic springs, added masses, or adjacent dummy structures at the boundary may be introduced. Adjacent substructures may be represented by their I/F stiffness and transformed masses (e.g., derived from a static condensation to the I/F DOFs). This is helpful if the number of generalized DOFs is limited, and an accurate approximation of the response of the uncondensed coupled structure shall be achieved where neither fixed nor free boundary conditions result in appropriate results. Moreover, the flexible boundary method allows for the introduction of mode shapes that reflect the influence of high damping. Because of the importance of the method for the damping treatment, the relationship of the flexible boundary method to other approaches is examined.

Relationship to Other Approaches

A powerful condensation tool is the mixed boundary method that deals with eigenmodes of the substructure where different DOFs of the t set are not fixed but free. One formulation has been published by Hintz [7]. The t set will be partitioned into the b set (DOFs fixed) and the c set (DOFs free). If constraint modes (C/M) associated with the complete t set are introduced, the matrix of Ritz vectors is

$$[G^*_{fa}]_{\text{C/M}} = \begin{bmatrix} I_{bb} & 0_{bc} & 0_{bq} \\ 0_{cb} & I_{cc} & \varphi_{cq} \\ \Phi_{ob} & \Phi_{oc} & \varphi_{oq} \end{bmatrix} \tag{15}$$

The flexible boundary method degenerates to the mixed boundary method if only clamped or free boundary conditions are taken into account. In this case, it is $[\varphi_{bq}] = [0_{bq}]$, and Eq. (11) becomes Eq. (15).

The mixed boundary method degenerates to the Craig–Bampton [1] approach if the c set is empty. The condensation method of Craig–Bampton demands for the constraint modes associated with the t set and the eigenmodes of the undamped system with the I/F to be fixed (t set clamped). The condensation method of Craig–Bampton is a more convenient formulation of the approach published by Hurty [8]. Both of the methods are equivalent. The Guyan method [9] may be regarded as a degeneration of the Craig–Bampton [1] approach with the q set empty. In this case, the t set has to include additional selected internal DOFs beside the coupling DOFs.

If constraint modes are associated with the b set, attachment modes associated with the c set may be added. For free structures that allow for rigid body modes (e.g., if the b set is empty), the attachment modes are introduced as inertia relief attachment modes. To determine these modes, a set of statically determined constraints is applied to the free matrix $[K_{ff}]$; the corresponding rows and columns will be put in the r set, and the remaining set is named d set:

$$[K_{ff}] = \begin{bmatrix} K_{dd} & K_{dr} \\ K_{rd} & K_{rr} \end{bmatrix} \tag{16}$$

The inverse of $[K_{dd}]$ is the flexibility matrix $[B_{dd}]$ that will be expanded to the flexibility matrix $[B_{ff}]$:

$$[B_{ff}] = \begin{bmatrix} B_{dd} & 0_{dr} \\ 0_{rd} & 0_{rr} \end{bmatrix} = \begin{bmatrix} K^{-1}_{dd} & 0_{dr} \\ 0_{rd} & 0_{rr} \end{bmatrix} \tag{17}$$

The inertia relief attachment modes are the columns of the matrix $[A_{ff}]$ with

$$[A_{ff}] = [P_{ff}]^T [B_{ff}][P_{ff}] \tag{18}$$

where the projection matrix $[P_{ff}]$ is defined by

$$[P_{ff}] = [I_{ff}] - [M_{ff}][\Phi_{fr}][\Phi_{fr}]^T \tag{19}$$

where $[\Phi_{fr}]$ are the rigid body modes that shall be normalized to the unit generalized mass. The multiplication of $[P_{ff}]^T$ with an arbitrary displacement results in an elimination of the rigid body motion of this displacement. Moreover, the multiplication of $[P_{ff}]$ with an arbitrary force yields a superposition of this force with inertia loads from a rigid body motion such that the resulting vector represents a self-equilibrating force. Thus, left-hand and right-hand multiplication of the flexibility $[B_{ff}]$ yields an inertia relief formulation of the flexibility. The inertia relief attachment modes degenerate to ordinary attachment modes if the fixed b set prevents rigid body motions. For an empty r set, the projection matrix is the identity matrix. Now, the matrix of Ritz vectors that includes the attachment modes (A/M) may be noted as

$$[G^*_{fa}]_{\text{A/M}} = \begin{bmatrix} I_{bb} & 0_{bc} & 0_{bq} \\ \Phi^*_{cb} & A_{cc} & \varphi_{cq} \\ \Phi^*_{ob} & A_{oc} & \varphi_{oq} \end{bmatrix} \tag{20}$$

Another approach may be formulated by introduction of the residual flexibility. Residual flexibility was first introduced in condensation techniques by Rubin and MacNeal (see, for example, [10,11]).

The difference between the complete flexibility in Eq. (18) and the flexibility represented by the retained eigenvectors $[\varphi_{fq}]$ of the free and undamped structure is the residual flexibility:

$$[R_{ff}] = [A_{ff}] - [\varphi_{fq}]\left[\text{diag}\left(\frac{1}{\omega_i^2}\right)\right][\varphi_{fq}]^T \tag{21}$$

where ω_i^2 is the eigenvalue of the ith mode. The eigenvectors shall be normalized to the unit generalized mass. Equation (21) makes clear that the residual flexibility is just a linear combination of attachment modes and retained eigenmodes; therefore, the introduction of attachment modes and eigenmodes as Ritz vectors is sufficient: no benefit is given by the residual flexibility. In spite of that, the matrix of Ritz vectors including the residual flexibility (R/F) will be noted as

$$[G^*_{fa}]_{\text{R/F}} = \begin{bmatrix} I_{bb} & 0_{bc} & 0_{bq} \\ \Phi^*_{cb} & R_{cc} & \varphi_{cq} \\ \Phi^*_{ob} & R_{oc} & \varphi_{oq} \end{bmatrix} \tag{22}$$

The residual flexibility is used by Majed et al. [12]. The transformation of the matrix from Eq. (22) with $[T_{aa}]$ from Eq. (5) directly yields the formulation of the transformation matrix used in [12].

It should be pointed out that all three formulations in Eqs. (15), (20), and (22) are equivalent. The Ritz vectors within the matrices span exactly the same space and will yield exactly the same results of the condensed equation of motion.

This may be demonstrated by the transformation with quadratic and nonsingular matrices $[H_{aa}]$. It may be derived from mode definition that

$$[\Phi_{oc}][A_{cc}] = [A_{oc}] \tag{23}$$

$$[\Phi_{ob}] + [\Phi_{oc}][\Phi^*_{cb}] = [\Phi^*_{ob}] \tag{24}$$

With this relation, it is

$$[G^*_{fa}]_{\text{A/M}} = [G^*_{fa}]_{\text{C/M}}[H_{aa}]_1 \tag{25}$$

and

$$[G^*_{fa}]_{\text{R/F}} = [G^*_{fa}]_{\text{A/M}}[H_{aa}]_2 = [G^*_{fa}]_{\text{C/M}}[H_{aa}]_1[H_{aa}]_2 \tag{26}$$

where

$$[H_{aa}]_1 = \begin{bmatrix} I_{bb} & 0_{bc} & 0_{bq} \\ \Phi^*_{cb} & A_{cc} & 0_{cq} \\ 0_{qb} & 0_{qc} & I_{qq} \end{bmatrix} \tag{27}$$

and

$$[H_{aa}]_2 = \begin{bmatrix} I_{bb} & 0_{bc} & 0_{bq} \\ 0_{cb} & I_{cc} & 0_{cq} \\ 0_{qb} & -[\text{diag}(1/\omega_i^2)][\varphi_{cq}]^T & I_{qq} \end{bmatrix} \tag{28}$$

The multiplication with $[H_{aa}]$ represents a linear transformation of the coordinates and does not change the space of the matrices. Therefore, the adequate formulation of the Ritz vectors for the mixed boundary method is given by Eq. (15). The other formulations complicate the calculations and yield no benefit.

Several specialized methods published in the past are available within the framework of the generalized method based on the transformation matrix in Eq. (8). The methods of Goldman [13] and Hou [14] are based on free eigenmodes only; b set and c set are empty. Craig [3] and Chang [15] improved these methods by adding the residual flexibility. The b set is empty, and the c set is associated with the residual flexibility. Another progress is given by the methods of Craig [3] and Chang [15] with the improvement of the MacNeal approach [11] by a Ritz formulation that minimizes the estimation error. However, the coupling DOFs are generalized. Martinez et al. [16] and Link [17] formulated the approach of Craig [3] and Chang [15] more conveniently with preserved physical DOFs at the I/Fs.

Gladwell [18] introduced a so-called branch mode analysis, where adjacent substructures are coupled as rigid bodies or clamped. Benfield and Hruda [19] used a so-called component mode substitution, where adjacent substructures may be represented by their I/F stiffness. Meirovitch and Hale [20] employed so-called admissible functions: for example, polynomials with a low degree.

Condensation of Damped Structures

All the condensation techniques discussed previously are based on eigenmodes of the undamped structures. If the mode shapes are influenced by damping effects (i.e., for high damping or nonproportional damping), the eigenmodes are complex. The homogeneous part of the solution of the differential equation of motion with the complex eigenvectors $\{\theta_f\} = \{\varphi_f\} + i\{\psi_f\}$, the eigenvalues $\lambda = \mu + i\omega$, and an arbitrary scalar q is

$$\{x_f\} = \{\theta_f\} q e^{(\mu + i\omega)t} = (\{\varphi_f\} + i\{\psi_f\}) q e^{(\mu + i\omega)t} \tag{29}$$

The combination of the complex and the conjugated complex solution yields

$$\{x_f\} = 2q(\{\varphi_f\} e^{\mu t} \cos \omega t - \{\psi_f\} e^{\mu t} \sin \omega t) \tag{30}$$

The superposition of the two modes $\{\varphi\}$ and $\{\psi\}$ results in moving zeros of the response. It is obvious that the displacement response of the structure will be a linear combination of the real parts and the imaginary parts of the complex eigenvectors. Therefore, the real Ritz vectors will be taken from both. The set of complex eigenvectors is noted as

$$[\theta_{fq}] = [\varphi_{fq}] + i[\psi_{fq}] \tag{31}$$

The generalized approach for complex eigenmodes (C/E) formulated by the flexible boundary method as described by Eqs. (11) and (14) results in a real matrix:

$$[G^*_{fa}]_{\mathrm{C/E}} = \begin{bmatrix} I_{bb} & 0_{bc} & \varphi_{bq} & \psi_{b\hat{q}} \\ 0_{cb} & I_{cc} & \varphi_{cq} & \psi_{c\hat{q}} \\ \Phi_{ob} & \Phi_{oc} & \varphi_{oq} & \psi_{o\hat{q}} \end{bmatrix} = \begin{bmatrix} I_{tt} & \varphi_{tq} & \psi_{t\hat{q}} \\ \Phi_{ot} & \varphi_{oq} & \psi_{o\hat{q}} \end{bmatrix} \tag{32}$$

where $\hat{q}$ denotes a set with additional generalized DOFs associated with the selected imaginary parts. In spite of complex modes, the complete condensation and coupling process remains within the real space.

As for the mixed boundary method, the modes are not inherently linear independent. The matrix should be checked and purged from linear dependencies if applicable (see previous).

Often, the real parts and the imaginary parts of the complex eigenvectors of the damped system may be represented approximately by a linear combination of the real undamped eigenvectors. In this case, a convenient condensation of the damped substructures is the condensation by matrices derived for the undamped system.

Coupling of Damped Substructures

Usually, in order to introduce modal damping on the substructure level, a damping force proportional to the generalized velocity is added to the equation of motion that is decoupled for a fixed t set:

$$\{F_D(\Omega)\} = i\Omega \begin{bmatrix} 0 & 0 \\ 0 & \mathrm{diag}(2\vartheta_i m_i \omega_i) \end{bmatrix} \begin{Bmatrix} x_t \\ q_q \end{Bmatrix}$$

where ϑ_i is the critical damping ratio of mode i, ω_i is the corresponding eigenfrequency, and m_i is the generalized mass. However, in most cases, the results obtained by this approach for the coupled structures are not reasonable. It is known that, for all systems composed from substructures with conventional modal damping, the system damping of the coupled system depends on the choice of the I/Fs. Moreover, even for constant modal damping of all the substructures, due to the coupling process, strange shifts of damping values are observed and damping coupling will occur. In [6], it is demonstrated that these effects are not a consequence of the coupling process or the condensation method, but they are a consequence of the representation of modal damping by viscous damping.

The approach that overcomes these problems is called the equivalent structural damping approach (ESD approach). Modal damping that is proportional to the modal velocity may be replaced by equivalent complex structural damping. The equation of motion for modal damping of a single DOF system is

$$-\Omega^2 mx + i\Omega dx + kx = F(\Omega) \tag{33}$$

The damping coefficient is $\vartheta = d/(2\sqrt{mk})$. The damping term will be replaced by an equivalent complex structural damping. For a single DOF system it is

$$-\Omega^2 mx + ic_{\mathrm{eq}} x + kx = F(\Omega) \tag{34}$$

Complex structural damping and modal damping result in the same resonance responses if

$$c_{\mathrm{eq}} = 2\vartheta k \tag{35}$$

It is well known that the response behavior of systems with modal damping is very close to the response behavior of systems with equivalent structural damping. Analogous for a multiple DOF system, it is

$$-\Omega^2[M]\{x\} + i\Omega[D]\{x\} + [K]\{x\} = \{F(\Omega)\} \tag{36}$$

and

$$-\Omega^2[M]\{x\} + i[C]_{\mathrm{eq}}\{x\} + [K]\{x\} = \{F(\Omega)\} \tag{37}$$

The equivalent structural damping matrix becomes

$$[C]_{\mathrm{eq}} = 2\vartheta[K] \tag{38}$$

Damping of structures consists of two parts. The first part is the damping induced by physical elements that may be viscous or have inherent structural material damping. The behavior of those elements must be known and idealized in a proper form. The ESD approach does not affect these terms within the equation of motion.

The second part is given by the modal damping ϑ_i associated with certain modes or by the standard system damping ϑ_0 associated with complete structures, referred to as constant modal damping or diagonal system damping. Usually, this part is idealized by viscous damping. Here, the ESD approach introduces the equivalent structural damping. It is essential for the ESD approach that all the damping is introduced on the substructure level, even if substructures are associated with the standard system damping only.

1) If no special damping behavior of a substructure is known, the substructure will be associated with the standard system damping ϑ_0. The structural damping matrix for this substructure will be derived from the stiffness matrix multiplied by twice the standard system damping:

$$[C_{aa}]_{\mathrm{eq}} = 2\vartheta_0[K_{aa}] \tag{39}$$

2) If dedicated modal damping ϑ_i for certain modes is known, and the equation of motion is decoupled for a fixed t set, it is

$$[C_{aa}]_{\mathrm{eq}} = \begin{bmatrix} 2\vartheta_0 K_{tt} & 0 \\ 0 & \mathrm{diag}(2\vartheta_i k_i) \end{bmatrix} \tag{40}$$

where k_i is the generalized stiffness with $k_i = m_i\omega_i^2$. For all modes without dedicated damping, the diagonal system damping must be taken into account: $\vartheta_i = \vartheta_0$.

If a damping matrix $[C_{aa}]$ exists from structural damping of dedicated physical elements or materials, the equivalent structural damping matrix $[C_{aa}]_{\mathrm{eq}}$ will be added. Then, the coupling of the mass, stiffness, and damping matrices of the substructures will be performed in the conventional manner. After coupling, no additional damping shall be applied.

Analysis of the Coupled Damped Structure

The coupled structure will be condensed before performing the transient or frequency response analysis. Now, the c set of the coupled structure is chosen to be empty, and the b set shall include all the DOFs that are really clamped for the response analysis; therefore, the uncondensed formulation of the equation of motion may be noted as

$$-\Omega^2[\hat{M}_{oo}]\{q_o\} + i\Omega[\hat{D}_{oo}]\{q_o\} + ([\hat{K}_{oo}] + i[\hat{C}_{oo}])\{q_o\} = \{F_o(\Omega)\} \tag{41}$$

The hat [ˆ] denotes the matrices of the coupled system. It should be pointed out that the matrix $[\hat{C}_{oo}]$ includes all the equivalent damping matrices of the substructures. No additional damping will be applied.

In case of high damping, the generalized condensation method with the matrix $[G^*_{fa}]$ from Eq. (32) is an appropriate tool. Of course, this transformation will condense but not decouple the equation of motion:

$$-\Omega^2[\hat{M}_{qq}]\{q_q\} + i\Omega[\hat{D}_{qq}]\{q_q\} + ([\hat{K}_{qq}] + i[\hat{C}_{qq}])\{q_q\} = \{F_q(\Omega)\} \tag{42}$$

A convenient condensation method for small damping is the condensation with the modes $[\varphi_{oq}]$ of the undamped system:

$$[\hat{M}_{oo}][\varphi_{oq}][\mathrm{diag}(\omega_i^2)] = [\hat{K}_{oo}][\varphi_{oq}] \tag{43}$$

with the eigenvalues ω_i^2. If the mode shapes are influenced by damping effects, it is of significant importance that the real parts and the imaginary parts of the complex eigenvectors of the damped system are approximately a linear combination of the real undamped eigenvectors. Introducing the eigenvectors of the undamped system, it becomes

$$-\Omega^2[\mathrm{diag}(m_i)]\{q_q\} + i\Omega[\hat{D}_{qq}]\{q_q\} + i[\hat{C}_{qq}]\{q_q\} + [\mathrm{diag}(k_i)]\{q_q\} = \{F_q(\Omega)\} \tag{44}$$

where

$$[\mathrm{diag}(m_i)] = [\varphi_{oq}]^T[\hat{M}_{oo}][\varphi_{oq}] \tag{45}$$

$$[\hat{D}_{qq}] = [\varphi_{oq}]^T[\hat{D}_{oo}][\varphi_{oq}] \tag{46}$$

$$[\hat{C}_{qq}] = [\varphi_{oq}]^T[\hat{C}_{oo}][\varphi_{oq}] \tag{47}$$

$$[\mathrm{diag}(k_i)] = [\varphi_{oq}]^T[\hat{K}_{oo}][\varphi_{oq}] \tag{48}$$

with the generalized mass m_i and the generalized stiffness k_i. Mass and stiffness matrices are decoupled, and the transformed damping matrices are fully populated.

If convenient, the condensed complex equation of motion (44) may be replaced by a formulation that allows an easy transformation to the time domain. This step will be called "transition to real space". At least for high responses, structural damping and modal damping are equivalent near the resonances ω_i, where the quotient Ω/ω_i is near to one. Replacing the term $i[\hat{C}_{qq}]\{q_q\}$ in Eq. (44) by

$$i\Omega[\hat{C}_{qq}]\left[\mathrm{diag}\left(\frac{1}{\omega_i}\right)\right]\{q_q\}$$

the complex equation of motion in the frequency domain may be approximated by the real formulation in the time domain:

$$[\mathrm{diag}(m_i)]\{\ddot{q}_q\} + [\hat{D}_{\mathrm{FTMP}}]\{\dot{q}_q\} + [\mathrm{diag}(k_i)]\{q_q\} = \{F_q(t)\} \tag{49}$$

with a modified FTMP:

$$[\hat{D}_{\mathrm{FTMP}}] = [\hat{D}_{qq}] + [\hat{C}_{qq}]\left[\mathrm{diag}\left(\frac{1}{\omega_i}\right)\right] \tag{50}$$

In a final step, the offdiagonal terms of the matrix $[\hat{D}_{\mathrm{FTMP}}]$ will be ignored. This step will be called "dropping the offdiagonal terms". It was observed that the neglect of the damping coupling was acceptable without essential loss of accuracy for most of the investigated systems, even for high damping. The equation of motion (49) decouples, and it becomes, for each mode i,

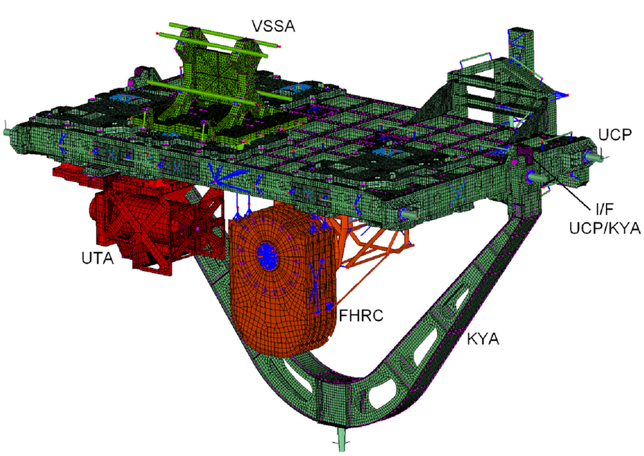

Fig. 1 FE model of the ICC.

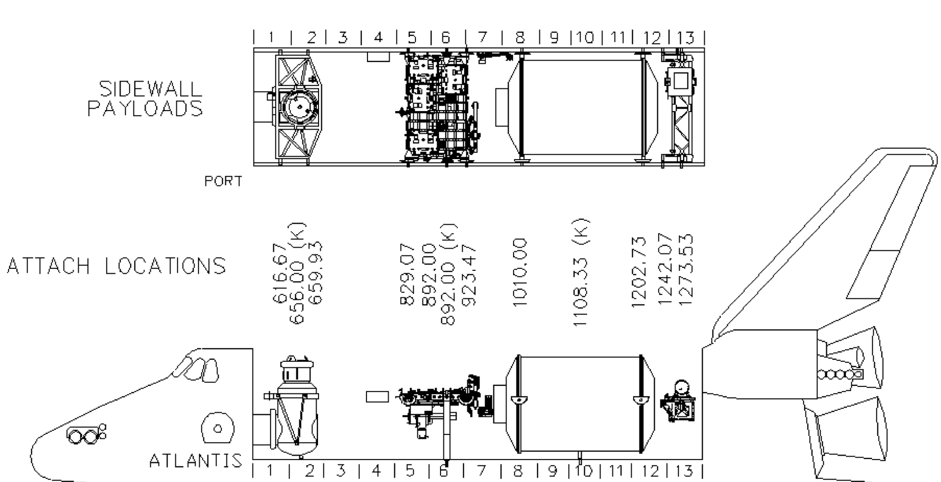

Fig. 2 ICC within the space shuttle at cargo bay position 5/7.

$$m_i\ddot{q}_i + d_i\dot{q}_i + k_iq_i = F_i(t) \tag{51}$$

with the diagonal terms d_i of the matrix $[\hat{D}_{\mathrm{FTMP}}]$.

In any case, the dropping of the offdiagonal terms shall be justified by comparison with the full formulation. A thumb rule that indicates the relevance of the offdiagonal terms is not known and may be a subject matter of future investigations.

Examples

The Integrated Cargo Carrier (ICC) is an unpressurized platform capable of carrying different payloads used in space shuttle missions to the International Space Station (ISS). The ICC was flown the first time onboard Space Shuttle Discovery during mission STS-96 in 1999. Meanwhile, the ICC has been in operation on 12 further space shuttle missions.

A finite-element (FE) model of the ICC, with integrated cargo, representing the flight configuration as it was planned for STS-114, is plotted in Fig. 1. Among other cargos, the video stanchion support assembly (VSSA), the utility transfer assembly (UTA), and the flex hose rotary coupling (FHRC) are mounted onto the ICC for the ISS supply mission. The ICC itself consists of the unpressurized cargo platform (UCP) and the keel yoke assembly (KYA), building the I/F to the space shuttle vehicle (SSV).

The ICC for flight STS-114 was integrated at cargo bay position 5/7, as shown in Fig. 2. Figure 3 shows a photograph of the space shuttle on orbit, taken by the ISS crew.

To demonstrate the application of the ESD approach, frequency response calculations are performed on the ICC as integrated for mission STS-114. Different damping characteristics have been studied for the presented results. A second frequency response calculation with the additional damping system OASIS (oscillation attenuation system for ICC services) is performed in order to demonstrate the effect of dropping the offdiagonal damping terms. Finally, modal damping characteristics for the complete shuttle mission STS-114 during liftoff are investigated, taking into account some modifications of the damping behavior.

The results of the following examples can be achieved by the flexible boundary method, as well as by the Craig–Bampton approach, with an appropriate increase of the number of DOFs. The examples focus on the treatment of damping.

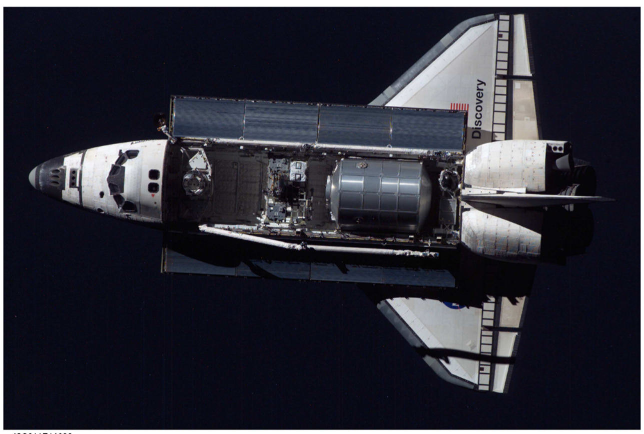

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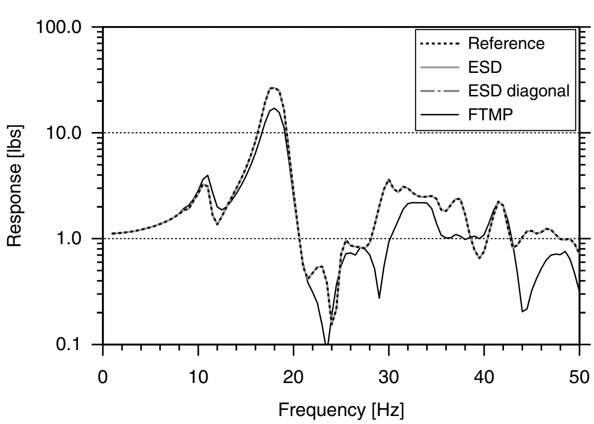
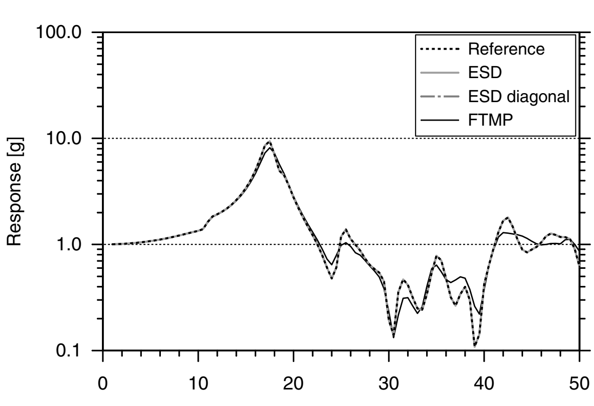
Fig. 3 Photograph of the mission.

First Example: Integrated Cargo Carrier with payload for Mission STS-114

For this configuration, among other cargos, VSSA, UTA, and FHRC are integrated using a passive frame adapter plate (PFAP). For a reference calculation with an uncondensed model, structural

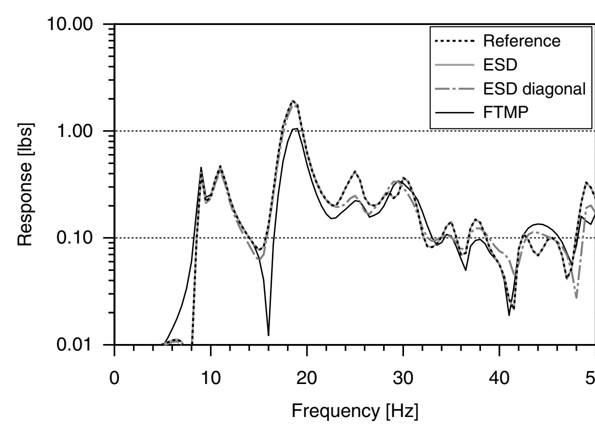


a) Force of a fastener screw PFAP/UCP

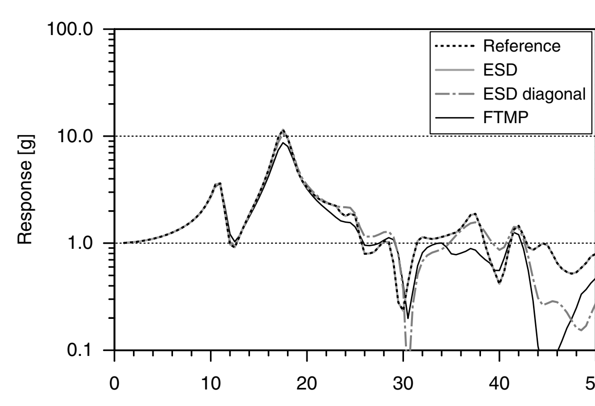


b) UTA center of gravity acceleration

Fig. 4 First example: ICC with payload for mission STS-114.



a) Force between UCP and KYA



b) VSSA center of gravity acceleration

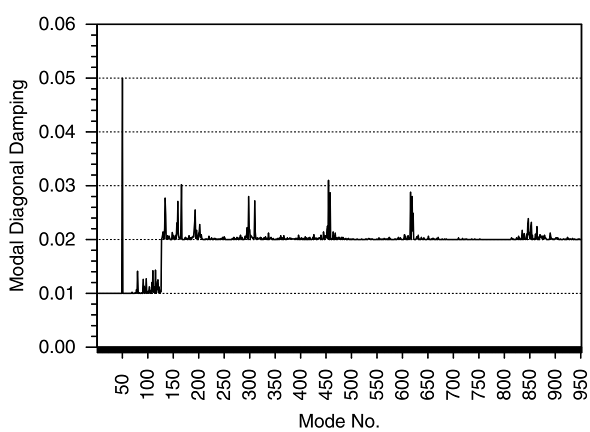
Fig. 5 Second example: ICC with payload and OASIS.

damping was introduced as material damping for each FE of each substructure: loss factor 0.10 (equivalent to 5% of the critical damping) for the UCP itself, loss factor 0.04 (equivalent to 2%) for the payload UTA, and loss factor 0.06 (equivalent to 3%) for the payload VSSA. In addition, an overall system damping of 1% was taken into account in the frequency response analysis. Following the ESD approach, all the damping, including the overall system damping, was introduced on the substructure level in terms of equivalent structural damping. The results of the ESD approach include the transition to real space as necessary for time domain analysis. For the classical FTMP approach, the dedicated substructure damping was introduced as modal damping for the condensed substructures. The overall system damping was taken into account for the coupled structure in the frequency response analysis.

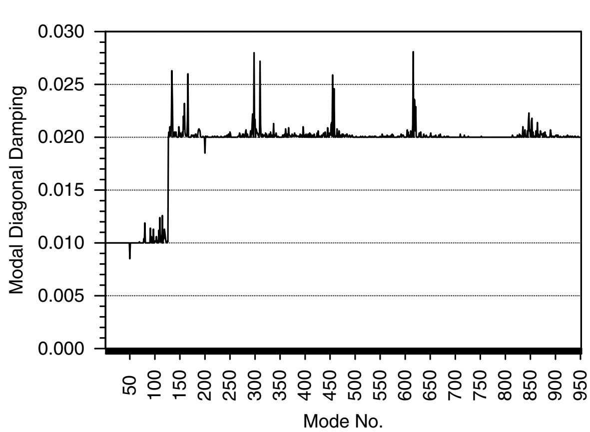
In Fig. 4a, the frequency response of the force of a fastener screw joining PFAP (with the mounted FHRC) and UCP is plotted. In Fig. 4b, the I/F force of the UTA (in terms of acceleration of the center of gravity) is plotted. The results of the reference calculations with the uncondensed model are the target for the coupled condensed models. The results of the ESD approach meet the reference results in an excellent manner. Moreover, if the offdiagonal damping terms are dropped, no visible deviations occur. The curve is denoted by "ESD diagonal". The classical FTMP approach results in severe errors.

Second Example: Integrated Cargo Carrier with Payload and OASIS

Additionally, the damping system OASIS is mounted. The OASIS was not used in the SSV missions; this configuration is investigated for study reasons only. The damping system works like a spring/damper system with an own overall damping with a loss factor of 0.4 (this corresponds to 20% modal damping). The other properties are the same as for the first calculation.



a) ESD approach



b) FTMP approach

Fig. 6 Third example: Modal diagonal damping for STS-114.

In Fig. 5a, the force acting between the UCP and the KYA is plotted. The response of the VSSA in terms of acceleration of the center of gravity is plotted in Fig. 5b. As observed for the first calculation, the ESD approach, including the transition to the real space, works very well. However, in contrast to the first example without OASIS, the dropping of the offdiagonal damping terms results in relevant deviations from the correct results. This example emphasizes that the evaluation of the offdiagonal terms is necessary. Again, the classical FTMP approach results in severe errors.

Third Example: Shuttle Mission STS-114

The modal damping as taken from the diagonal terms of the coupled damping matrix of the complete space shuttle mission are calculated for the liftoff of mission STS-114 up to 35 Hz. Free modes of the space shuttle with mounted ICC shall have a modal damping of 1% for frequencies smaller than 10 Hz and 2% for higher frequencies. A modal damping of 5% is allocated to the mathematical model of the remote manipulator system (RMS) of the space shuttle. In addition, 5% damping is allocated to the major mode of the ICC.

The damping coefficients are plotted in Fig. 6. These coefficients are taken from the diagonal terms of the real damping matrices; coupling terms are ignored. For the ESD approach, the expected behavior is found. The mode 50 is the major ICC mode and has the expected damping of 5%. However, the FTMP approach yields only 0.85% for this mode. Moreover, the introduction of higher-damped components should yield an increase but no decrease of the damping of the coupled structure. This behavior is found by the ESD approach but not by the classical FTMP approach. The FTMP approach results in a damping value of less than 2% for mode 200 of the coupled structure.

Conclusions

The generalized condensation technique, completed by the ESD approach, provides a powerful tool for condensation, coupling, and analyzing of structures. The problems associated with the coupling of damped structures are overcome.

The generalized condensation technique simplifies the condensation of structures. Three steps are necessary: the choice of appropriate Ritz vectors, the transformation of these vectors, and the condensation. The result is a consistent formulation of condensation techniques for undamped and damped structures, including high damping and nonproportional damping. It was demonstrated that all the condensation methods that minimize the approximation error may be represented within the framework of the generalized approach. It was found that the introduction of attachment modes or residual flexibility provides no benefit and is unnecessary. Therefore, a flexible boundary method is introduced that is based on constrained modes and a set of arbitrary mode shapes. This formulation allows for the special demands of substructures, even with high and nonproportional damping, and supplies the condensation process with additional potentialities.

The ESD approach overcomes severe problems that may arise with the coupling of structures with different component damping. The damping is introduced as equivalent structural damping on substructure level. The transition of the complex equation of motion to the real space is possible. Moreover, decoupling of the equation of motion is possible for certain conditions.

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